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Comment on "Numerical Lifting-Surface Theory—Problems and Progress"

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IN a recent paper, Landahl and Stark[1] present a progress report on the status of numerical approaches to nonsteady lifting-surface theory for planar and nonplanar configurations as applied to the linearized thin-wing problem, with particular stress on the subsonic case.

Over the past few years, in the course of studies adapting unsteady lifting-surface theory to marine propellers,[2–4] Davidson Laboratory has developed a new method for the solution of the downwash surface integral equation. By proper expansion of the kernel function and introduction of the so-called "generalized lift operator," the chordwise integration is performed analytically with the additional advantage that the numerical solution is greatly simplified. These studies indicate that use of the generalized lift operator, which is in fact dictated by the nature of the integral equation itself, is a more accurate and rapid procedure than the "usual" numerical approaches for evaluating the steady and unsteady pressure distributions on lifting surfaces and resultant hydrodynamic forces.

This technique has been used in Ref. 5, where the lifting surfaces are the blades of a marine propeller operating in nonuniform inflow, and in Ref. 6 for the case of a deeply submerged, flat, rectangular hydrofoil in steady flow. In Ref. 7, this new approach has been applied to several two-dimensional, unsteady airfoil problems and has yielded results identical to the known explicit solutions.

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Reply by Authors to S. Tsakonas

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WE are grateful to S. Tsakonas for having drawn our attention to the interesting developments in lifting-surface theory at Davidson Laboratory. The successful application of the generalized lift operator, a simplified version of the variational method,[1] seems very valuable. Such an analytic chordwise integration is of course desirable also in the case of oscillating aircraft wings. Therefore, it should be valuable to know whether a similar quadrature process could be developed for compressible flow as well. There is one apparently unsettled question, however, namely that of the poor convergence of the lift distributions shown in Ref. 2.

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Comment on "Transonic Flow in Unconventional Nozzles"

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AN effort to derive the equations published in an analysis by Hopkins and Hill[1] pertaining to the transonic flow regime in annular and other unconventional nozzles has yielded some discrepancies. We believe the equations are correct as written in the following. The numbering and the nomenclature of the equations correspond to the numbering and nomenclature of the equations in the original paper.

$$(H/H_R)^2 = 1 + (M_R{}^2q_1 - q_1)\Delta\eta + [q_1{}^2 - q_2 + (M_R{}^2/2)(2q_2 - q_1{}^2) + (\gamma/2)M_R{}^4q_1{}^2]\Delta\eta^2 + [2q_1q_2 - q_1{}^3 + (M_R{}^2/2)(q_1{}^3 - 2q_1q_2 + 2q_3) + \gamma M_R{}^4q_1q_2]\Delta\eta^3 \quad (18)$$

$$x = \xi - \frac{H_R{}^2\eta_R{}^2}{2Y_R{}^3}(2H_RH_R'Y_R - H_R{}^2\sin\omega)\Delta\eta^2 + \left[\frac{H_R{}^4\eta_R{}^3}{3Y_R{}^5}\cos\omega\left(4H_RH_R'Y_R - \frac{5}{2}H_R{}^2\sin\omega\right) - \frac{H_R{}^3H_R'\eta_R}{Y_R{}^2} + \frac{H_R{}^4\eta_R\sin\omega}{2Y_R{}^3}\right]\Delta\eta^3 \quad (20)$$

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$$y = \frac{H_R^2\eta_R}{Y_R}\,\Delta\eta - \left(\frac{H_R^4\eta_R^2\cos\omega}{2Y_R^3} - \frac{H_R^2}{2Y_R}\right)\Delta\eta^2 + \Bigg[\frac{H_R^6\eta_R^3\cos^2\omega}{2Y_R^5} - \frac{H_R^4\eta_R\cos\omega}{2Y_R^3} + \frac{4}{3}\,\frac{H_R^5H_R'\eta_R^3\sin\omega}{Y_R^4} - \frac{H_R^6\eta_R^3\sin^2\omega}{2Y_R^5} - \frac{H_R^4(H_R')^2\eta_R^3}{Y_R^3} - \frac{H_R^5H_R''\eta_R^3}{3Y_R^3} + M_R^2\left(\frac{H_R^5H_R''\eta_R^3}{3Y_R^3} + \frac{H_R^4(H_R')^2\eta_R^3}{3Y_R^3} - \frac{2H_R^5H_R'\eta_R^3\sin\omega}{3Y_R^4} + \frac{H_R^6\eta_R^3\sin^2\omega}{3Y_R^5}\right)\Bigg]\Delta\eta^3 \tag{21}$$

$$\frac{M^*}{M_R^*} = 1.0 + \frac{H_R^2\eta_R^2}{Y_R^4}\,[H_RH_R''Y_R^2 + (H_R')^2Y_R^2 - 2H_RH_R'Y_R\sin\omega + H_R^2\sin^2\omega]\Delta\eta^2 + \Bigg[\frac{H_R^2(H_R')^2\eta_R}{Y_R^2} + \frac{H_R^3H_R''\eta_R}{Y_R^2} + \frac{H_R^4\eta_R\sin^2\omega}{Y_R^4} - \frac{8H_R^4(H_R')^2\eta_R^3\cos\omega}{3Y_R^4} - \frac{4H_R^5H_R''\eta_R^3\cos\omega}{3Y_R^4} - \frac{2H_R^3H_R'\eta_R\sin\omega}{Y_R^3} + \frac{16H_R^5H_R'\eta_R^3\cos\omega\sin\omega}{3Y_R^5} - \frac{8H_R^6\eta_R^3\sin^2\omega\cos\omega}{3Y_R^6}\Bigg]\Delta\eta^3 \tag{22}$$

$$\theta = \frac{H_R^2\eta_R}{Y_R^2}\left(\frac{2H_R'Y_R}{H_R} - \sin\omega\right)\Delta\eta + \Bigg[\frac{H_R^2\eta_R^2\cos\omega}{Y_R^4}\times\left(\frac{3H_R^2\sin\omega}{2} - 2H_RH_R'Y_R\right) + \frac{H_RH_R'}{Y_R} - \frac{H_R^2\sin\omega}{2Y_R^2}\Bigg]\Delta\eta^2 + \Bigg[-\frac{2H_R^3H_R'\eta_R\cos\omega}{Y_R^3} + \frac{3H_R^4\eta_R\sin\omega\cos\omega}{2Y_R^4} - \frac{5H_R^6\eta_R^3\cos^2\omega\sin\omega}{2Y_R^6} + \frac{3H_R^5H_R'\eta_R^3\cos^2\omega}{Y_R^5} - \frac{2H_R^3(H_R')^3\eta_R^3}{3Y_R^3} - \frac{5H_R^4H_R'H_R''\eta_R^3}{3Y_R^3} + \frac{8H_R^4(H_R')^2\eta_R^3\sin\omega}{3Y_R^4} - \frac{10H_R^5H_R'\eta_R^3\sin^2\omega}{3Y_R^5} + \frac{4H_R^5H_R''\eta_R^3\sin\omega}{3Y_R^4} + \frac{4H_R^6\eta_R^3\sin^3\omega}{3Y_R^6} - \frac{H_R^5H_R'''\eta_R^3}{3Y_R^3} + M_R^2\left(\frac{4H_R^3(H_R')^3\eta_R^3}{3Y_R^3} - \frac{13H_R^4(H_R')^2\eta_R^3\sin\omega}{3Y_R^4} + \frac{7H_R^4H_R'H_R''\eta_R^3}{3Y_R^3} + \frac{14H_R^5H_R'\eta_R^3\sin^2\omega}{3Y_R^5} - \frac{5H_R^5H_R''\eta_R^3\sin\omega}{3Y_R^4} - \frac{5H_R^6\eta_R^3\sin^3\omega}{3Y_R^6} + \frac{H_R^5H_R'''\eta_R^3}{3Y_R^3}\right) + M_RM_R'\left(\frac{2H_R^4(H_R')^2\eta_R^3}{3Y_R^3} + \frac{2H_R^5H_R''\eta_R^3}{3Y_R^3} - \frac{4H_R^5H_R'\eta_R^3\sin\omega}{3Y_R^4} + \frac{2H_R^6\eta_R^3\sin^2\omega}{3Y_R^5}\right)\Bigg]\Delta\eta^3 \tag{23}$$

$$A = \pi(Y^2 - Y_R^2)/\cos\omega \tag{28}$$

$$H_R^2 = \{[1 + C_1(1 - e^{[-(\xi-S)^2/2R_SC_1]})]^2\cos\omega + 2[1 + (C_1 - C_1e^{[-(\xi-S)^2/2R_SC_1]})][Y_0 + \xi\sin\omega]\}/\{[1 + C_1(1 - e^{-S^2/2R_SC_1})]^2\cos\omega + 2[1 + C_1(1 - e^{-S^2/2R_SC_1})]Y_0\} \tag{35}$$

The discrepancies in Eqs. (18) and (21) occur in the coefficients of the $\Delta\eta^2$ terms. Equations (20–23) yielded discrepancies in the coefficients of the $\Delta\eta^3$ terms.

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Comments on "Effects of a Dynamic Gas on Breakdown Potential"

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GARDNER has presented some experimental results[1] showing that as the flow velocity in a channel is increased, the breakdown potential decreases from its no-flow value and then becomes nearly constant. We shall present an argument that explains, qualitatively, these data.

Since breakdown of a gas is said to occur when the electron density reaches a certain value, the classical calculation of breakdown essentially amounts to an accounting of the electron loss and production mechanisms. Forced convection appears to be a further loss mechanism; consequently the dynamic gas effect is expected to result in an increase rather than a decrease in the breakdown potential. The actual situation is somewhat more complicated for two reasons. First, the electric field in Gardner's experiment seems to be nonuniform, being larger near the electrodes (which constitute two walls of the channel) than near the center of the channel. Second, the distribution of the electrons is different in windoff and windon situations. Recent calculations[2] have shown that, in the presence of forced convection near electrically conducting walls, the electron density profile is fuller; that is, there is a greater electron density. That this is true follows from the fact that forced convection increases electron transport to the wall (as well as heat transfer, etc.). The electron transport is equal to the product of the diffusion coefficient and the normal gradient of number density. Since the diffusion coefficient is constant, the increase in transport is due to the increased gradient, which in turn implies that the profile must be fuller in a convecting fluid.

The production of electrons over a fixed distance is proportional to the product of the number density of electrons with the number of ionizing collisions per electron in this distance. [This term is essentially Townsend's first ionization coefficient (α).] The basic problem is to calculate the electric potential required to increase the electron density to some fixed level. The nonuniform field is such that α is largest near the electrode. Thus, forced convection offers the possibility of reducing breakdown potential by increasing the number of electrons in a high-voltage region as long as the increased losses due to forced convection do not increase as fast as the rate of production of electrons increases. Reference 2 shows that when the electromotive force decreases quadratically with distance away from the wall, the effect of forced convection is to reduce the breakdown voltage around a slot antenna. A detailed calculation for the direct current case depends upon the electric field distribution but also upon

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